Analysis of a Aircraft Landing Gear Suspension System

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Abstract

The objective of this paper was to determine current system attributes of landing gear on a runway, to develop a new landing gear system which reduces vibration at $200km/hr$, and to analyze the robustness of the new system by analytically testing it on a sawtooth surface. We determined the current system attributes by analyzing the transient response of the system using logarithmic decrement. We found that the current system attributes were $k = 4418000 {}^{N}\!/_{m}$ and $c = 25549 \frac{Ns}{m}$ Then, we developed the new landing gear system by methodically altering the system's damping coefficient c and spring constant k such that the peak amplitude occurred after the specified range of operation. Our newly designed spring constant and damping coefficient were $5743400 {}^{N}\!/_{m}$, and $51098 {}^{Ns}\!/_{m}$, respectively. These are reasonable because they are similar to the real world original values of the system. Finally, we generated the frequency spectrum of the newly designed system on a sawtooth runway in order to determine the robustness of the design. The frequency spectrum was then analyzed to determine if the natural frequency of the system matched with any of the harmonics. We found that the natural frequency did not match any of the harmonics, thus we concluded that this is a robust system. We conclude the paper with possible suggestions for further tests of our system's robustness and possible areas of further study.

1 Introduction & Problem Analysis

The landing gear of an airplane may be modeled as the spring-mass-damper system as shown in Figure 1 (a). The runway surface is described by the curve $y(t) = Y_0 \cos \omega t$ with a surface amplitude of $Y_0 = 0.1$ meters and a wavelength of $\lambda = 12$ meters. The mass of the landing gear is 2000 kilograms. The accompanying free body diagram is Figure 1 (b).

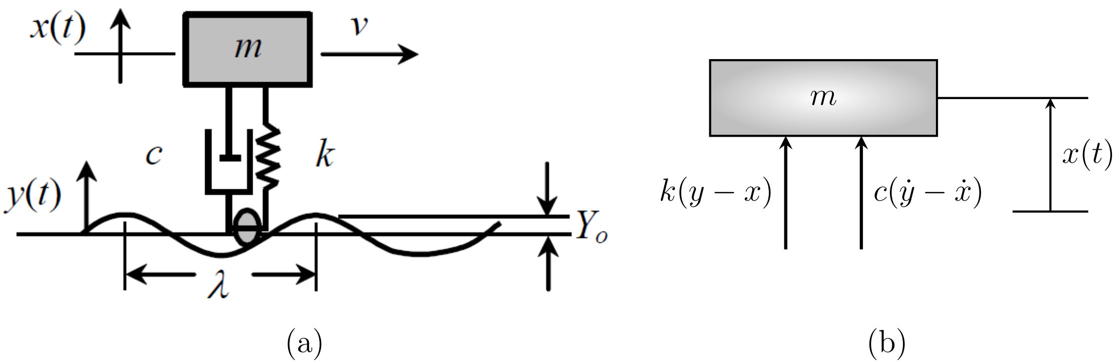


Figure 1: (a) Landing gear model and (b) free body diagram.

To identify the existing system parameters, an experiment was performed to capture the time trace of the response. The transient motion is recorded in Figure 2.

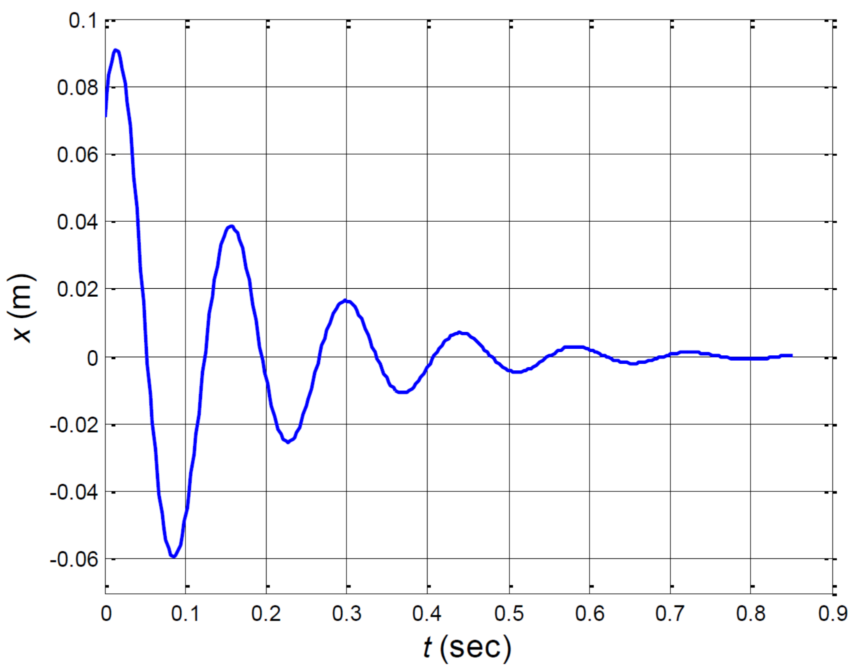


Figure 2: Transient response of existing landing gear system.

1.1 Analysis of the Runway Surface

We know that the runway surface can be modeled by the curve $y(t) = Y_0 \cos \omega t$. Where $Y_0 = 0.1$ meters, and $\omega = {}^{2\pi\nu}/_{\lambda}$, where ν is the horizontal velocity of the system, and $\lambda = 12$ meters. Thus we can resolve the curve to a simpler form of $y(t) = 0.1 \cos \frac{2\pi\nu}{12} t$.

1.2 Transient Analysis

To find the damping damping ratio, ζ, we will use logarithmic decrement. We take x_1 and x_2 at the first two positive peaks in Figure 2. We approximate their values to be $x_1 = 0.09m$ and $x_2 = 0.038m$. We also take their time value, which we approximate to be $t_1 = 0.02s$ and $t_2 = 0.155s$. We use these two values to calculate $\delta = \ln {}^{x_1}/_{x_2} = \ln 0.09/0.038 = 0.862$. We also know that $\delta = \zeta\omega_n\tau_d$, which we can further simplify to:

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

Now solving for ζ we get that:

$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}} = \frac{0.862}{\sqrt{(2\pi)^2 + (0.862)^2}} = .13595$$

Knowing the damping ratio for the current landing gear system allows us to determine the frequency response by varying the ratio ${}^{\omega}/_{\omega_n}$ and plotting the resulting amplitude, $|\widetilde{X}|$, and phase angle, ϕ. A detailed derivation of this is given in Appendix A, and the results are presented in Figure 3.

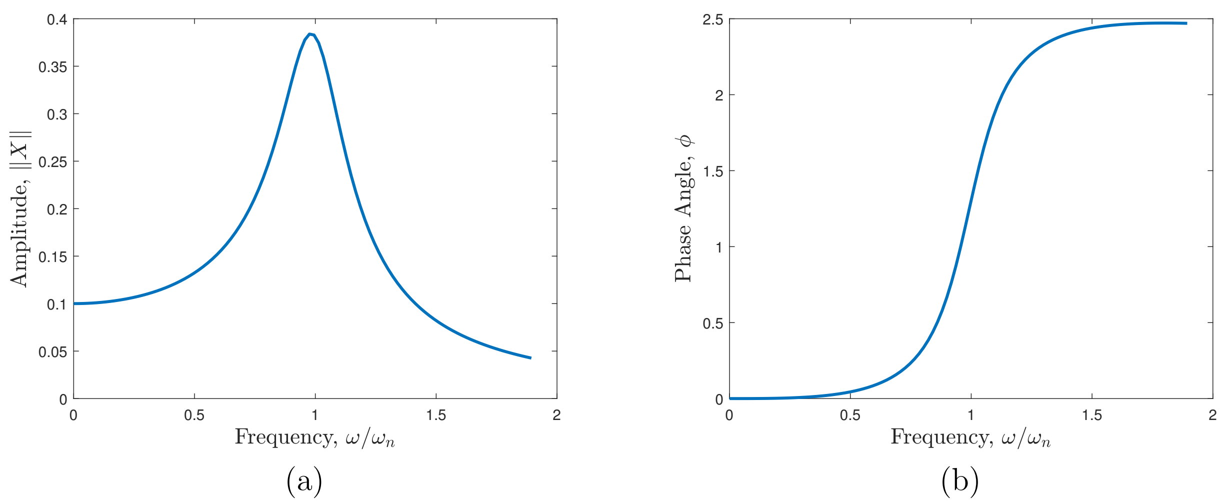


Figure 3: (a) Amplitude Response and (b) Phase Angle Response for existing design.

We will use the time values of t_1 and t_2 to calculate τ_d:

$$\tau_d = t_2 - t_1 = 0.155 - 0.02 = 0.135s$$

Now we use the original equation, $\delta = \zeta\omega_n\tau_d$, to find ω_n, which we then use to kind k for our system:

$$\begin{aligned}\omega_n &= \frac{\delta}{\zeta\tau_d} \\ &= \frac{0.8622}{(0.1359)(0.135)} \\ &= 47\end{aligned}$$

Now we use the definition of ω_n to calculate k:

$$\begin{aligned}\omega_n &= \sqrt{k/m} \\ k &= \omega_n^2 m \\ &= (47)^2(2000) \\ &= 4,418,000 = 4.418 \times 10^6 N/m\end{aligned}$$

Finally we use the original definition of ζ to find c:

$$\begin{aligned}\zeta &= \frac{c}{2m\omega_n} \\ c &= 2\zeta m\omega_n \\ &= 2(0.1359)(2000)(47) \\ &= 25,549.2\ Ns/m\end{aligned}$$

The derivation of the steady state response is presented in Appendix A. Three cycles of steady-state motion are presented in Figure 4.

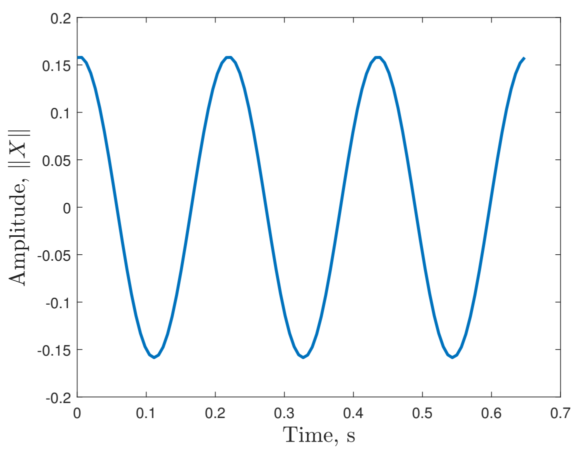
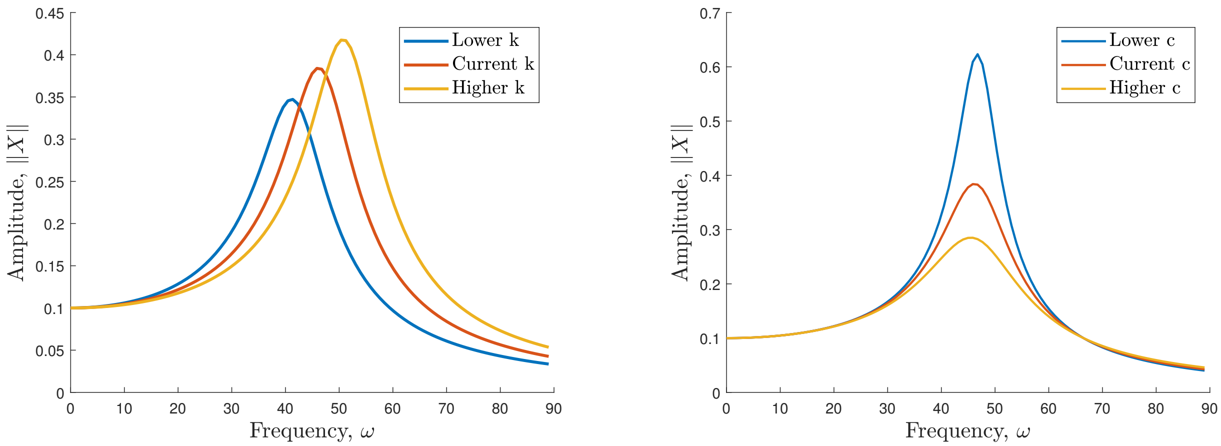


Figure 4: Steady state response for current landing gear system.

2 Simulation & Results

2.1 Design Selection

Because it is a concern at $200^{km}/_{hr}$, it is desired that the amplitude of vibration be limited to $0.145m$ at this velocity. We wish to design the spring, k, and damper, c, to ensure such motion. In order to optimize the response of the system, we have to understand how damper and stiffness control work. To do this, we hold k constant while varying c and hold c constant while varying k. The frequency responses of these two actions are shown in Figure 5 (a), and (b).



(a) (b)

Figure 5: (a) Amplitude Response (Variable k) and (b) Amplitude Response (Variable c).

From this behavior, we observe that increasing c decreases the peak amplitude and increasing k shifts the peak to the right. For the operating range of 0 to $200^{km}/_{hr}$, we want to lower and shift the peak amplitude such that it occurs after $200^{km}/_{hr}$. This strategy is chosen because the current system's peak amplitude is to the right of the frequency at $200^{km}/_{hr}$. Therefore, smaller changes to the current design are required to push the peak further to the right instead of pulling it significantly to the left. Keeping the optimized design close to the current parameters increases manufacturability which will be discussed more later. With this desired result in mind, we increase k and c until this condition is met. The designs leading up to this final design are shown in Figure 6.

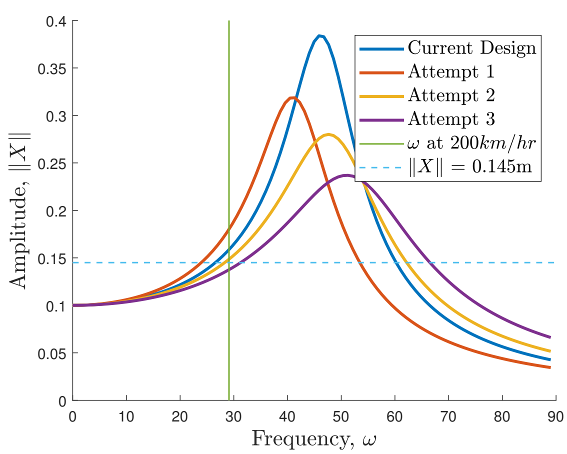
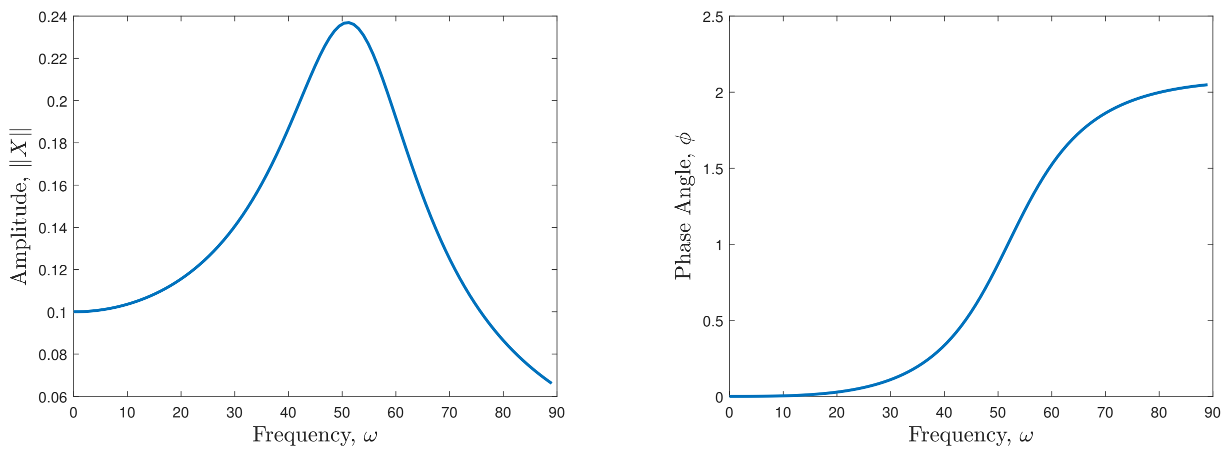


Figure 6: Frequency response of current system and three designs leading to the chosen design.

The frequency response for our proposed solution, Attempt 3, is presented in Figure 7. This solution has values of $k = 5743400 N/m$, $c = 51098 Ns/m$, and $\omega_n = 53.58$.

To ensure this design is within the preference for an underdamped system, we calculate the damping ratio, $\zeta = 0.2384$, and confirm that the system is underdamped.

Finally, we consider the feasibility of this design in terms of spring and damper specifications. When adjusting the spring constant, we stop at a 30% nominal increase to ensure our design is manufacturable. The damping of the system can be increased by adding damping material, so this value is used to push the amplitude below the maximum specification at $200 km/hr$.



(a) (b)

Figure 7: (a) Amplitude Response and (b) Phase Angle Response for solution.

2.2 Analysis of Design Robustness

Suppose that along the runway while the airplane is at 200 km/hr, there is an unforeseen change in the surface, described by Figure 8. This square wave surface continues for 6 cycles.

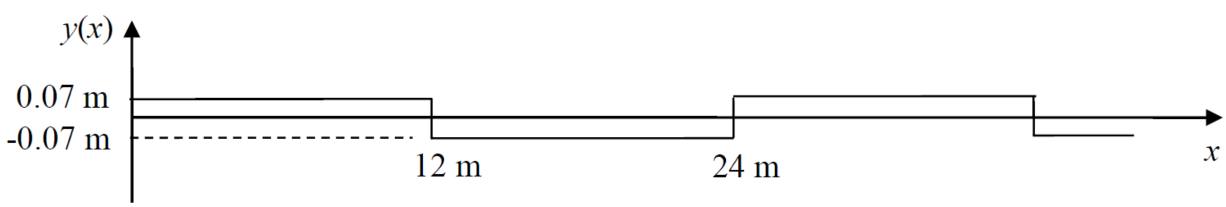


Figure 8: Unforeseen motion of the base.

This square wave may be expanded as a Fourier series, as described in Appendix B. Truncating the series at 11 terms produces the approximation shown in Figure 9.

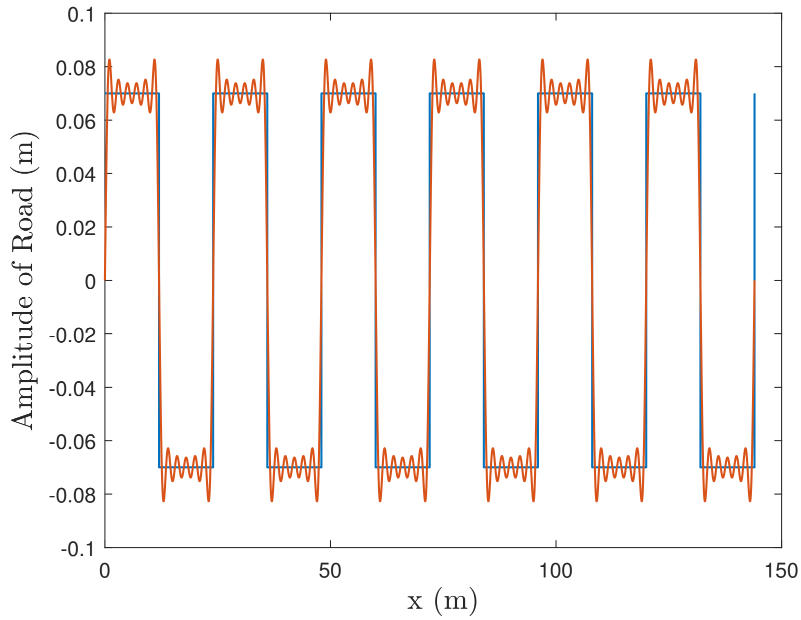


Figure 9: Fourier approximation of square wave with 11 Fourier terms.

We note that $\omega_0 = 2\pi/\lambda$, where $\lambda = 24$ m for the inconsistency in the runway surface. We check to see if our proposed natural frequency $\omega_n = 53.58$ is a harmonic and find that $\omega_n/\omega_0 \approx 204.6$. Since the natural frequency is not a harmonic frequency, no component should be a concern. However, the 205$^{\text{th}}$ comes closest.

3 Conclusions

The purpose of this project was threefold; to determine the current response of a landing gear system, to develop a new landing gear system to reduce vibration at $200^{km}/_{hr}$, and to analyze the robustness of the solution by considering the response of the system to a saw tooth surface. The original system was found to exceed the desired maximum amplitude of $0.145m$ at $200^{km}/_{hr}$. The new system was chosen so that the peak amplitude occurs after the range of operation and does not exceed the desired amplitude. Our designed spring constant, $k = 5743400^{N}/_{m}$, and damping coefficient, $c = 51098^{Ns}/_{m}$, are similar to the real world nominal values ensuring feasibility of implementation.
This work could be extended by testing more runway shapes to ensure a solution that works on many different surfaces. Additionally, the design selection process can be expanded to consider more factors, such as the smoothness of the ride. For example, this could be approached as an optimization problem where one would define a loss function related to the magnitude of the derivative of the amplitude curve and seek physically possible solutions that minimize the maximum derivative.

Appendix A Steady State Response derivation

To derive $|\widetilde{X}|$ and ϕ we must first find the Equation of Motion of the system. Thus we follow the four steps to producing an Equation of Motion bellow:

1. Select a suitable coordinate to describe motion:
   We choose $x(t)$, the motion of the mass, to describe the motion of the system

2. Determine the inertia reference:
   We choose towards the top of the page to be the positive direction, and we want to evaluate the system at its equilibrium position.

3. Displace the system and draw the Free Body Diagram:
   Now we displace the system such that $y > x$. And the free body diagram is displayed in figure 1(b).

4. Apply Newton's 2^{nd} law of motion to the Free Body Diagram:

$$\begin{aligned} m\ddot{x} &= \sum F_x \\ m\ddot{x} &= c(\dot{y} - \dot{x}) + k(y - x) \end{aligned}$$

   Solving, we find that our Equation of Motion is:

$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky \tag{1}$$

Using our equation of motion, we will derive the frequency response.

$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$

We know that $y(t) = Y_0 \cos \omega t$, and thus $\dot{y}(t) = -\omega Y_0 \sin \omega t$. Now we assume a harmonic solution so that we can subsitute $y(t) = Y_0 e^{i\omega t}$, and $\dot{y}(t) = i\omega Y_0 e^{i\omega t}$. Plugging this back into the EOM we get:

$$m\ddot{x} + c\dot{x} + kx = c(i\omega Y_0 e^{i\omega t}) + k(Y_0 e^{i\omega t})$$

Define a new variable: $\widetilde{Y} = (k + ic\omega)Y_0$.
Thus we can write a more familiar equation:

$$m\ddot{x} + c\dot{x} + kx = \widetilde{Y} e^{i\omega t}$$

We now assume a solution ($x(t) = \widetilde{X} e^{i\omega t}$) and plug this back in the EOM:

$$\begin{aligned} m(-\omega^2 \widetilde{X} e^{i\omega t}) + c(i\omega \widetilde{X} e^{i\omega t}) + k(\widetilde{X} e^{i\omega t}) &= \widetilde{Y} e^{i\omega t} \\ \widetilde{X} e^{i\omega t}[-m\omega^2 + ic\omega + k] &= \widetilde{Y} e^{i\omega t} \end{aligned}$$

$$\widetilde{X} = \frac{\widetilde{Y}}{(k - m\omega^2) + ic\omega}$$

Plugging this back into $x(t)$, and recalling that we defined $\widetilde{Y} = (k + ic\omega)Y_0$ we get that:

$$x(t) = \frac{(k + ic\omega)Y_0}{(k - m\omega^2) + ic\omega} e^{i\omega t}$$

Notice that both the denominator and numerator of the fraction in the equation above are complex numbers, and thus can be converted into polar form:

$$x(t) = \frac{Y_0\sqrt{k^2 + (c\omega)^2}e^{i\phi_1}}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}e^{i\phi_2}}e^{i\omega t}$$

From our knowledge of imaginary numbers we know that: $\phi_1 = \tan^{-1}\frac{c\omega}{k}$ and $\phi_2 = \tan^{-1}\frac{c\omega}{k-m\omega^2}$. Define a new variable $\phi = \phi_2 - \phi_1$. Now through our knowledge of trigonometry identities and the small angle assumption we get that:

$$\phi = \tan^{-1}\frac{cm\omega^3}{k(k - m\omega^2) + (c\omega)^2}$$

But our solution is still in the complex plane, and we want a solution that is either entirely real. And since our prescribed motion is $y(t) = Y_0 \cos\omega t)$, we want the real part of the complex solution:

$$x_p(t) = \mathrm{Re}\{|\widetilde{X}|e^{i(\omega t - \phi)}\} = |\widetilde{X}|\cos\omega t - \phi$$

Finally we will represent $|\widetilde{X}|$ and ϕ in terms of ω_n and ζ:

$$\begin{aligned}
|\widetilde{X}| &= Y_0\sqrt{\frac{1 + (2\zeta^{\omega}/_{\omega_n})^2}{[1 - (^{\omega}/_{\omega_n})^2]^2 + [2\zeta^{\omega}/_{\omega_n}]^2}} \\
\phi &= \tan^{-1}\frac{2\zeta(^{\omega}/_{\omega_n})^3}{1 - (^{\omega}/_{\omega_n})^2 + (2\zeta^{\omega}/_{\omega_n})^2}
\end{aligned}$$

Appendix B Fourier Transform of Square Wave

Upon inspection we find that the function is both symmetric about the x-axis and is odd we know that $a_0 = 0$, and that $a_j = 0$ for all $j \in \mathbb{N}$. A short proof of this is given below:

$$\begin{aligned}
a_0 &= \frac{2}{\tau}\int_0^{\tau} y(x)dx \\
&= \frac{2}{24}\left[\int_0^{12} .07dx + \int_{12}^{24} -.07dx\right] \\
&= \frac{1}{12}\left[.07(12 - 0) - .07(24 - 12)\right] \\
a_0 &= 0
\end{aligned}$$

$$\begin{aligned}
a_j &= \frac{2}{\tau}\int_0^{\tau} y(x)\cos(j\omega_0 x)dx \\
&= \frac{2}{24}\left[\int_0^{12} .07\cos(j\omega_0 x)dx + \int_{12}^{24} -.07\cos(j\omega_0 x)dx\right] \\
&= \frac{1}{12}\left[\frac{.07}{\omega_0 j}(\sin 12j\omega_0 - \sin 0) + \frac{-.07}{\omega_0 j}(\sin 24j\omega_0 - \sin 12j\omega_0)\right] \\
&= \frac{.07}{12\omega_0 j}\left[\sin(\pi j) - 0 - \sin(2\pi j) + \sin(\pi j)\right] \\
&= \frac{.07}{12\omega_0 j}[0] \\
a_j &= 0
\end{aligned}$$

Given that both of these terms go to zero we know that the fourier transform of this square wave will be of the form:

$$f(x) = \sum_{j=1}^{\infty} b_j \sin(j\omega t)$$

So, next we must solve for b_j:

$$\begin{aligned}
b_j &= \frac{2}{\tau}\int_0^{\tau} y(x)\sin(j\omega_0 x)dx \\
&= \frac{1}{12}\left[\int_0^{12} .07\sin(j\omega_0 x)dx + \int_{12}^{24} -.07\sin(j\omega_0 x)dx\right] \\
&= \frac{1}{12}\left[\frac{-.07}{\omega_0 j}(\cos(12j\omega_0) - \cos 0) + \frac{.07}{\omega_0 j}(\cos(24j\omega_0) - \cos(12j\omega_0))\right] \\
&= \frac{.07}{12j\omega_0}\left[-\cos(\pi j) + 1 + \cos(2\pi j) - \cos(\pi j)\right] \\
&= \frac{.07}{\pi j}\left[1 + \cos(2\pi j) - 2\cos(\pi j)\right] \\
b_j &= \frac{.07}{\pi j}\left[2 - 2(-1)^j\right]
\end{aligned}$$

Thus we can conclude that the fourier transform of the square wave given in part 4 of the project is equivalent to:

$$f(x) = \sum_{j=1}^{\infty}\left[\frac{.07}{\pi j}(2 - 2(-1)^j)\sin j\frac{2\pi}{24}t\right]$$